

9.6 Cosmology

Cosmological model parameters

Hubble law	$v_r = Hd$	(9.83)	v_r radial velocity H Hubble parameter d proper distance
Hubble parameter ^a	$H(t) = \frac{\dot{R}(t)}{R(t)}$ $H(z) = H_0 [\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + (1 - \Omega_{m0} - \Omega_{\Lambda 0})(1+z)^2]^{1/2}$	(9.84) (9.85)	0 present epoch R cosmic scale factor t cosmic time z redshift
Redshift	$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{R_0}{R(t_{\text{em}})} - 1$	(9.86)	λ_{obs} observed wavelength λ_{em} emitted wavelength t_{em} epoch of emission
Robertson–Walker metric ^b	$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$	(9.87)	ds interval c speed of light r, θ, ϕ comoving spherical polar coordinates
Friedmann equations ^c	$\ddot{R} = -\frac{4\pi}{3}GR \left(\rho + 3\frac{p}{c^2} \right) + \frac{\Lambda R}{3}$ $\dot{R}^2 = \frac{8\pi}{3}G\rho R^2 - kc^2 + \frac{\Lambda R^2}{3}$	(9.88) (9.89)	k curvature parameter G constant of gravitation p pressure Λ cosmological constant
Critical density	$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$	(9.90)	ρ (mass) density ρ_{crit} critical density
Density parameters	$\Omega_m = \frac{\rho}{\rho_{\text{crit}}} = \frac{8\pi G\rho}{3H^2}$ $\Omega_\Lambda = \frac{\Lambda}{3H^2}$ $\Omega_k = -\frac{kc^2}{R^2H^2}$ $\Omega_m + \Omega_\Lambda + \Omega_k = 1$	(9.91) (9.92) (9.93) (9.94)	Ω_m matter density parameter Ω_Λ lambda density parameter Ω_k curvature density parameter
Deceleration parameter	$q_0 = -\frac{R_0\ddot{R}_0}{\dot{R}_0^2} = \frac{\Omega_{m0}}{2} - \Omega_{\Lambda 0}$	(9.95)	q_0 deceleration parameter

^aOften called the Hubble “constant.” At the present epoch, $60 \lesssim H_0 \lesssim 80 \text{ km s}^{-1} \text{ Mpc}^{-1} \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, where h is a dimensionless scaling parameter. The Hubble time is $t_H = 1/H_0$. Equation (9.85) assumes a matter dominated universe and mass conservation.

^bFor a homogeneous, isotropic universe, using the $(-1, 1, 1, 1)$ metric signature. r is scaled so that $k = 0, \pm 1$. Note that $ds^2 \equiv (ds)^2$ etc.

^c $\Lambda = 0$ in a Friedmann universe. Note that the cosmological constant is sometimes defined as equalling the value used here divided by c^2 .



Cosmological distance measures

Look-back time	$t_{\text{lb}}(z) = t_0 - t(z)$	(9.96)	$t_{\text{lb}}(z)$ light travel time from an object at redshift z t_0 present cosmic time $t(z)$ cosmic time at z
Proper distance	$d_p = R_0 \int_0^r \frac{dr}{(1 - kr^2)^{1/2}} = cR_0 \int_t^{t_0} \frac{dt}{R(t)}$	(9.97)	d_p proper distance R cosmic scale factor c speed of light t_0 present epoch
Luminosity distance ^a	$d_L = d_p(1 + z) = c(1 + z) \int_0^z \frac{dz}{H(z)}$	(9.98)	d_L luminosity distance z redshift H Hubble parameter ^b
Flux density–redshift relation	$F(\nu) = \frac{L(\nu')}{4\pi d_L^2(z)}$ where $\nu' = (1 + z)\nu$	(9.99)	F spectral flux density ν frequency $L(\nu)$ spectral luminosity ^c
Angular diameter distance ^d	$d_a = d_L(1 + z)^{-2}$	(9.100)	d_a angular diameter distance k curvature parameter

^aAssuming a flat universe ($k=0$). The apparent flux density of a source varies as d_L^{-2} .

^bSee Equation (9.85).

^cDefined as the output power of the body per unit frequency interval.

^dTrue for all k . The angular diameter of a source varies as d_a^{-1} .

Cosmological models^a

	$d_p = \frac{2c}{H_0} [1 - (1 + z)^{-1/2}]$	(9.101)	d_p proper distance
Einstein – de Sitter model	$H(z) = H_0(1 + z)^{3/2}$	(9.102)	H Hubble parameter
($\Omega_k = 0$, $\Lambda = 0$, $p = 0$ and $\Omega_{m0} = 1$)	$q_0 = 1/2$	(9.103)	t_0 present epoch
	$t(z) = \frac{2}{3H(z)}$	(9.104)	z redshift
	$\rho = (6\pi Gt^2)^{-1}$	(9.105)	c speed of light
	$R(t) = R_0(t/t_0)^{2/3}$	(9.106)	q deceleration parameter
Concordance model	$d_p = \frac{c}{H_0} \int_0^z \frac{\Omega_{m0}^{-1/2} dz'}{[(1 + z')^3 - 1 + \Omega_{m0}^{-1}]^{1/2}}$	(9.107)	$t(z)$ time at redshift z
($\Omega_k = 0$, $\Lambda = 3(1 - \Omega_{m0})H_0^2$, $p = 0$ and $\Omega_{m0} < 1$)	$H(z) = H_0[\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})]$	(9.108)	R cosmic scale factor
	$q_0 = 3\Omega_{m0}/2 - 1$	(9.109)	Ω_{m0} present mass density parameter
	$t(z) = \frac{2}{3H_0} (1 - \Omega_{m0})^{-1/2} \text{arsinh} \left[\frac{(1 - \Omega_{m0})^{1/2}}{(1 + z)^{3/2}} \right]$	(9.110)	G constant of gravitation
			ρ mass density

^aCurrently popular.